MALAYSIA MATHEMATICAL OLYMPIAD 2007/2008

Malaysian Mathematical Sciences Society

Malaysia IMO 2008 Selection and Training Program

The Malaysia IMO 2008 Selection and Training Program is a yearlong program to identify talented Malaysian students to represent the country in IMO 2008. The program is organized by the Malaysian Mathematical Sciences Society (*Persatuan Sains Matematik Malaysia*, or PERSAMA), under the purview of the Ministry of Education, Malaysia.

Assoc. Prof. Dr. Arsmah Ibrahim, the Vice President of PERSAMA, served as the Chairperson of the PERSAMA IMO 2008 Committee. The training sessions were jointly organized by Assoc. Prof. Dr. Daud Mohamad (Leader, IMO 2008), Assoc. Prof. Dr. Jamaludin M. Ali (Leader, IMO 2007), Assoc. Prof. Dr. Mat Rofa Ismail (Leader, IMO 2005) and Suhaimi Ramly (Deputy Leader, IMO 2007).

National Mathematical Olympiad (23 June 2007)

The first selection test is the National Mathematical Olympiad (*Olimpiad Matematik Kebangsaan*, or OMK), which is open to all Malaysian high school students. A total of 7,872 students, distributed over 14 exam centers, took part in OMK 2007.

IMO Selection Test (3 November 2007)

The top 120 contestants from the OMK were selected to take part in the IMO Selection Test.

IMO Training Camp 1 (7–12 December 2007)

The top 30 contestants from the IMO Selection Test were invited to attend the centralized IMO Training Camp 1 at MARA Institute of Technology in Shah Alam. During the six-day camp, students were given lectures and daily exams, culminating in a Final Exam on the last day.

IMO Training Camp 2 & Asia Pacific Mathematics Olympiad (APMO) (7–11 March 2008)

From the first Training Camp, 17 students were further shortlisted to attend the IMO Training Camp 2. On the last day of this camp, the students wrote the APMO. Among them, three went on to win APMO awards: Loke Zhi Kin won a Silver, while Saw Yihui and Lim Yu Wei won a Bronze each.

IMO Training Camp 3 & Final Training Sessions (24–28 May 2008)

Ten shortlisted students were invited to attend the IMO Training Camp 3. The students were given daily IMO style exams as well as lectures on advanced topics. At the end of the camp, the Malaysia IMO 2008 team was finalized. They are:

LOKE Zhi Kin	Seafield HS, Selangor
SAW Yihui	USJ 12 HS, Selangor
LIM Yu Wei	Keat Hwa HS, Kedah
Muhamad Amir M. FADZIL	MARA College Banting, Selangor
Joshua LIM Kai Tsen	St. David's HS, Malacca
THAM Ying Hong	Catholic HS, Selangor

Throughout June and early July, the final training took place in the form of weekly meetings and correspondence exams.

Selected Problems from the Malaysia IMO 2008 Selection and Training Program

1. Find all pairs (a, b) of positive integers such that $a^4 + 1$ and $b^2 - 1$ are not divisible by 39 but $(a^4 + 1)(b^2 - 1)$ is.

2. Given an acute triangle ABC. Points A_1, B_1, C_1 lie on ray BC, CA and AB respectively, such that triangle $A_1B_1C_1$ is similar to triangle ABC. It is known that $A_1B = B_1C_1$. Show that $A_1B_1 \times C_1B + B_1C_1 \times A_1B = A_1C_1^2$.

3. Let $b, n > 1$ be integers. Suppose that for each $k > 1$ there exists an integer a_k such that $b - a_k^n$ is divisible by k. Prove that b is an nth power.

4. Let $n > m \geq 1$ be positive integers, and consider the set

$$M = \{(x, y) \mid x, y \in \mathbb{N}, 1 \leq x, y \leq n\}.$$

Determine the least value $v(m, n)$ with the property that for any subset $P \subseteq M$ with $|P| = v(m, n)$ there exist $m + 1$ elements $A_i = (x_i, y_i) \in P, i = 1, 2, \ldots, m + 1$, for which the values x_i are all distinct, and the values y_i are also all distinct.

Solution

1. Find all pairs (a, b) of positive integers such that a^4+1 and b^2-1 are not divisible by 39 but $(a^4+1)(b^2-1)$ is.

   Solution: All fourth powers are congruent to 0 or 1 (mod 3), so we have $a^4+1 \equiv 1$ or 2 (mod 3).

   Therefore, for $(a^4+1)(b^2-1)$ to be divisible by 39, 3 must divide b^2-1. But since 39 does not divide b^2-1, we have that 13 divides a^4+1.

   Here, we can check that a^4+1 can only be congruent to $1, 2, 4, 10 \pmod{13}$, so there are no solution.

   Alternatively, we note that $a^4+1 = 0 \pmod{13}$ implies $a^8 = 1 \pmod{13}$. By Fermat's Little Theorem, we have $a^{12} = 1 \pmod{13}$. Therefore, we can conclude that $a^{\gcd(8,12)} = 1 \pmod{13}$ or $a^4 = 1 \pmod{13}$, which contradicts the fact that $a^4 = -1 \pmod{13}$.

   (Solution by Joshua Lim)

2. Given an acute triangle ABC. Points A_1, B_1, C_1 lie on ray BC, CA and AB respectively, such that triangle $A_1B_1C_1$ is similar to triangle ABC. It is known that $A_1B = B_1C_1$. Show that $A_1B_1 \times C_1B + B_1C_1 \times A_1B = A_1C_1^2$.

   Solution: Notice that $\angle A_1BC_1 + \angle A_1B_1C_1 = 180°$. Reflect point B to point B' along side A_1C_1. Now the points A_1, B_1, C_1, B' are concyclic. By Ptolemy's theorem, $A_1B_1 \times B'C_1 + B_1C_1 \times A_1B' = A_1B_1 \times C_1B + B_1C_1 \times A_1B = A_1C_1 \times B_1B'$.

   Now it remains to be shown that $B_1B' = A_1C_1$. Let B_1B' and A_1C_1 intersect at point P. Thus

   $$\frac{B_1P}{A_1P} = \frac{B_1C_1}{A_1B'} = 1$$

   and

   $$\frac{B'P}{C_1P} = \frac{A_1B'}{B_1C_1} = 1.$$

   Therefore, $B_1B' = A_1P + C_1P = A_1C_1$.

   (Solution by Saw Yihui)

3. Let $b, n > 1$ be integers. Suppose that for each $k > 1$ there exists an integer a_k such that $b - a_k^n$ is divisible by k. Prove that b is an nth power.

   Solution: Let $k = b^2$. Then there are integers a_k and m such that $b - a_k^n = mb^2$, or $a_k^n = b(1-mb)$. The numbers b and $1-mb$ are relatively prime, and since their product is an nth power, each of the number is an nth power as well.

   (Solution by Loke Zhi Kin)

4. Let $n > m \geq 1$ be positive integers, and consider the set

   $$M = \{(x, y) \mid x, y \in \mathbb{N}, 1 \leq x, y \leq n\}.$$

Determine the least value $v(m,n)$ with the property that for any subset $P \subseteq M$ with $|P| = v(m,n)$ there exist $m+1$ elements $A_i = (x_i, y_i) \in P, i = 1, 2, \ldots, m+1$, for which the values x_i are all distinct, and the values y_i are also all distinct.

Solution: We claim that $v(m,n) = mn + 1$.

Partition M into n sets $P_k = \{(x,y) \mid x + y - k \equiv_n 0\}, k = 1, 2, \ldots, n$. The pigeonhole principle now forces at least $m+1$ elements from P to belong to a same P_k; let them be $A_i = (x_i, y_i)$. Now, if we assume $x_i = x_j$, then from $x_i + y_i - k \equiv_n x_j + y_j - k$ it follows that $y_i \equiv_n y_j$. But $y_i, y_j \in \{1, 2, \ldots, n\}$, it follows that $y_i = y_j$, i.e. $A_i = A_j$.

Conversely, $mn + 1$ is the least cardinality of P to warrant the claimed result; for if $|P| = mn$, one can pick $P = \{(x,y) \mid 1 \leq x \leq m, 1 \leq y \leq n\}$; then any $m+1$ elements from P, be them $A_i = (x_i, y_i)$, will share at least one $x_i = x_j$ (pigeonhole principle again).

(from the Romanian Selection Test for the Junior Balkan Mathematical Olympiad 2007. Solution from the Romanian Mathematical Competition 2007 booklet).

Malaysian Mathematical Sciences Society

The Malaysian Mathematical Sciences Society was established in 1970 by a group of mathematicians from Universiti Malaya to replace the defunct Malayan Mathematical Sciences Society. Currently the Society has more than 300 members, among them professional mathematicians, academics, teachers, students, and math enthusiasts.

The stated mission of the Malaysian Mathematical Sciences Society is to uphold mathematical sciences and its practitioners through excellence in articulation and sophistication, mathematical discovery, unity of knowledge and development of human capital.

The Society is involved in numerous mathematics related activity in the country. The Society publishes two serials for its members, "Bulletin Of The Malaysian Mathematical Society (Second Series)" and "*Menemui Matematik*" (Discovering Mathematics). Since 1996, the Society has been tasked by the Ministry of Education to select and train students for the IMO.

Official website: `www.persama.org.my`.

About our Sponsor

PETRONAS, the acronym for Petroliam Nasional Berhad, was incorporated on 17 August 1974. Over the years, PETRONAS has grown to become a fully-integrated oil and gas corporation and is ranked among FORTUNE Global 500's largest corporations in the world.

PETRONAS has four subsidiaries listed on the Bursa Malaysia and has ventured globally into more than 32 countries worldwide in its aspiration to be a leading oil and gas multinational of choice.

Official website: www.petronas.com.